Market Influence of Portfolio Optimizers

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What is influence on the market?

Question: Do investors that are portfolio optimizers influence the market in some way that is different from that of standard or reference traders?

Related question: Do traders that maintain hedging portfolios (program traders) influence the market in a particular way?

Also: What effect does statistical arbitrage have on markets?

Background

The effect of hedgers or program traders has been of interest at least since 1987. Theoretical studies appeared in the 90's (Föllmer and Schweitzer 93, Fray and Stremme 97, Sircar and Papanicolaou 98, Platten and Schweitzer 98, Schönbucher and Wilmott 00).

Main result (with a couple of different models), both with analysis and simulations:

Program traders tend to increase the volatility of the underlying asset.

Why is this so?

Calibration issues are difficult here, and largely unexplored.

Back to portfolio optimizers

Do portfolio optimizers influence the market?

Yes, and largely in a "good" way because they tend to decrease the volatility

Why do they tend to decrease volatility?

Analytical model outline

- Assume that there is an incomes process that together with the underlying asset price process determines the demand for the asset of the reference traders
- The portfolio optimizers maintain a portfolio made up from the risky asset and a risk free one by maximizing an expected utility of their wealth at a target time horizon
- The risky asset price process adjusts so as to satisfy the demand both from reference traders and from the portfolio optimizers through a clearing constraint
- This couples the evolution of the asset price process and the wealth process through the HJB equation for the value of the wealth and the clearing condition

Mathematical formulation (Appl Math Fin 08)

Aggregate incomes process:

$$dY_t = \mu(Y_t, t)dt + \eta(Y_t, t)dB_t,$$

with $\mu = \mu_1 y$ and $\eta = \eta_1 y$

Risky asset price process:

$$dX_t = \alpha X_t dt + \sigma X_t dB_t$$

Here α and σ are not known but are determined by the market clearing condition.

Riskless asset price:

$$d\beta_t = r\beta_t dt$$

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Demand of reference traders: $D(X_t, Y_t, t)$. We take $D(x, y) = f(y^\gamma/x)$ and eventually $f(z) \sim z$

The wealth process satisfies

$$dW_t = W_t(\pi(\alpha - r) + r)dt + W_t \pi \sigma dB_t$$

with $W_0 = w$. Self-financing is assumed.

The allocation fraction π is determined from the HJB equation

$$\begin{aligned} V_t + \sup_{\pi} \left[\frac{1}{2} V_{ww} \pi^2 \sigma^2 w^2 + V_w(\pi(\alpha - r) + r)w + V_{xw} \sigma^2 \pi x w \right] \\ + V_x \alpha x + \frac{1}{2} V_{xx} \sigma^2 x^2 = 0, \end{aligned} \tag{1}$$

for $t < T$ with $V(x, w, T) = u(w)$, that is satisfied by the optimal value function

$$V(x, w, t) = \sup_{\pi} E_{x,w,t} \left[u(W_T) \right]$$

with $u(w)$ a utility function.

Merton theory; portfolio optimizers demand

When the risky asset price process is log normal and the utility is $u(w) = w^\lambda/\lambda$, $0 < \lambda \leq 1$, then the HJB equation is explicitly solvable and the allocation fraction is independent of time:

$$\pi_0 = \frac{\alpha_0 - r}{\sigma_0^2(1-\lambda)}$$

We will assume that the demand of the portfolio optimizers is:

$$\Phi = \frac{\pi W_t}{X_t}$$

Therefore, assuming that the total shares in the risky asset are fixed we have the clearing condition

$$D(X_t, Y_t, t) + \Phi(X_t, W_t, t) \equiv S_0$$

Run through full interaction for ”one time cycle”

Start the incomes process Y_t.

Start with an evolution for X_t, that is, with an α and a σ.

Start with a fixed π (from the Merton theory).

Advance the wealth process W_t. Calculate new α and σ from the clearing condition.

Solve the HJB equation one step back. Get new allocation fraction.

Advance one more cycle.

Small interaction analysis

Assume that Φ, the demand of the portfolio optimizers is small.

Then the first order correction to a log normal model for X_t is, for the mean return (drift):

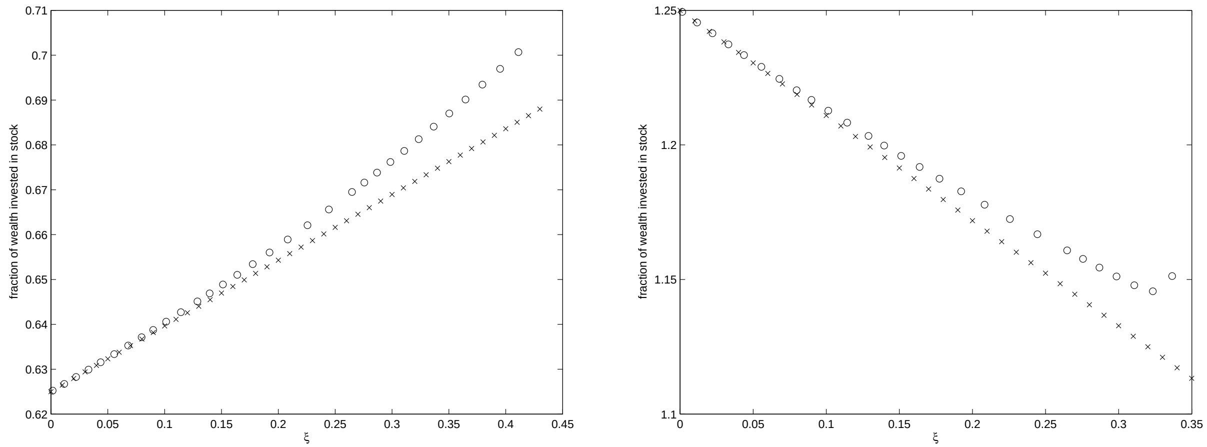
$$\alpha_1 = \frac{\pi_0 W_t}{X_t S_0}((\pi_0 - 1)(\alpha_0 - r))$$

and for the volatility:

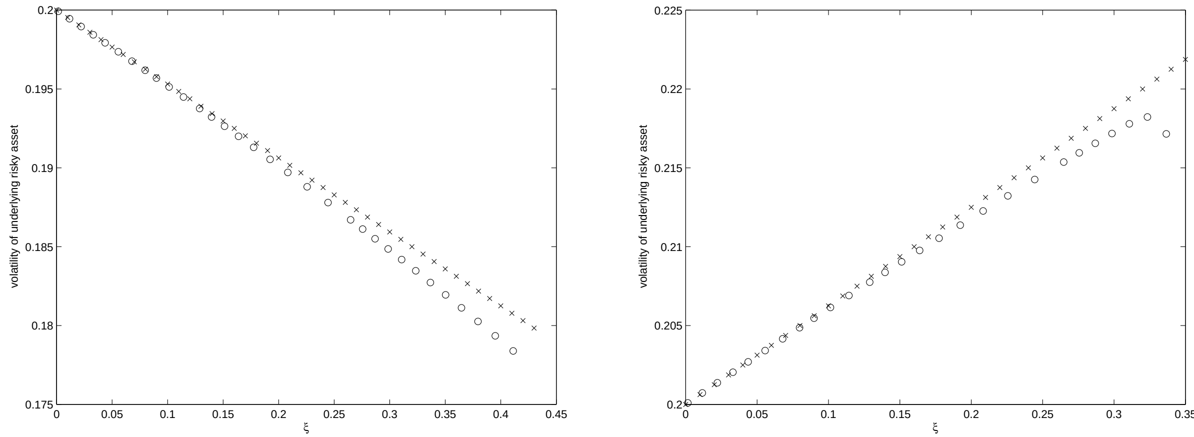
$$\sigma_1 = \sigma_0(\pi_0 - 1)\frac{\pi_0 W_t}{X_t S_0}$$

Note that $\sigma_1 < 0$ when $0 < \pi < 1$, the "normal" case.

Allocation fraction



Portfolio allocation to stock at time $t = 0$, when stock has low excess returns (left) and high excess returns (right): comparison between approximation ('x') and numerical solution of PDE ('o').



Local volatility of the stock at time $t = 0$, when stock has low excess returns (left) and high excess returns (right): comparison between approximation ('x') and numerical solution of PDE ('o').

Summary and conclusions

- Feedback effects can be estimated with "simple" macro market models. For example, both the effect of hedgers and the effect of portfolio optimizers can be assessed.

- Mathematics: very complex and highly nonlinear PDE problems already at the simplest level of modeling of feedback effects.

- Calibration issues are largely untouched. They are quite difficult. They could provide interesting information of market trends.

- What about statistical arbitrage?